BOROUGH OF MANHATTAN COMMUNITY COLLEGE
The City University of New York

Department of Mathematics

Title of Course: Bridge to Advanced Mathematics **Instructor:**
Course: MAT 310 **Tel #:**
Semester: **Office:**
Credits: 3 **Office hrs:**
Class hrs: 3 **Email:**

Course Description: This course is designed to prepare students for an advanced mathematics curriculum by providing a transition from Calculus to abstract mathematics. The course explores the logical and foundational structures of mathematics, with an emphasis on understanding and writing proofs. Topics include logic, methods of proof, mathematical induction, axiomatic approach to group theory, number theory, set theory, relations and functions, Cantor's theory of countability, and the development of the real number system. Throughout the course, students will be actively engaged in understanding, verifying, and writing proofs, and will be introduced to methods of mathematics research.

Corequisite: Calculus II (MAT 302) or Departmental approval.

Required Text: Chartrand, Gary; Polimeni, Albert; and Zhang, Ping, *Mathematical Proofs: A Transition to Advanced Mathematics* (2013). Pearson.

Student Learning Outcomes:

1) Students will be able to evaluate the validity of a proposed proof.
2) Students will be able to understand and use formal reasoning methods to create valid proofs.
3) Students will be able to prove mathematical theorems.
4) Students will be able to recognize the role of sets in mathematics.
5) Students will be able to use the foundational aspect of functions and relations.
6) Students will be able to apply axiomatic methods to make discoveries in abstract topics in mathematics.
7) Students will be able to produce undergraduate level research projects.

General Education Learning Outcomes	Measurements
Quantitative Reasoning. Students will be able to use quantitative skills and the concepts and methods of mathematics to solve problems.	Quizzes, tests, homework
Information & Technology Literacy. Students will be able to collect, evaluate and interpret information and effectively use information technologies.	Quizzes, tests, homework

Goals for all Curricula	Measurements
a. Students should be able to communicate their thoughts in an organized, coherent manner both orally and in writing.	Quizzes, tests, homework
b. Students should be able to analyze problems, identify probable causes and suggest possible solutions.	Quizzes, tests, homework
c. Students should acquire knowledge of the past and demonstrate an understanding of its relationship to the present and future.	Quizzes, tests, homework
d. Students should be able to transfer previous learning to new situations and demonstrate faculty for acquiring new knowledge.	Quizzes, tests, homework

Technology:

Evaluation and Requirements of Students: At the beginning of the semester, the instructor will advise the student how the final grade will be determined (based on class work, examinations, quizzes, writing assignments and the final examination).
Students are required to attend all scheduled classes.

College Attendance Policy

At BMCC, the maximum number of absences is limited to one more hour than the number of hours a class meets in one week. For example, you may be enrolled in a three-hour class. In that class, you would be allowed 4 hours of absence (not 4 days). In the case of excessive absences, the instructor has the option to lower the grade or assign an F or WU grade.

Academic Adjustments for Students with Disabilities

Students with disabilities who require reasonable accommodations or academic adjustments for this course must contact the Office of Services for Students with Disabilities. BMCC is committed to providing equal access to all programs and curricula to all students.

BMCC Policy on Plagiarism and Academic Integrity Statement

Plagiarism is the presentation of someone else's ideas, words or artistic, scientific, or technical work as one's own creation. Using the idea or work of another is permissible only when the original author is identified. Paraphrasing and summarizing, as well as direct quotations, require citations to the original source. Plagiarism may be intentional or unintentional. Lack of dishonest intent does not necessarily absolve a student of responsibility for plagiarism.

Students who are unsure how and when to provide documentation are advised to consult with their instructors. The library has guides designed to help students to appropriately identify a cited work. The full policy can be found on BMCC's web site, www.bmcc.cuny.edu. For further information on integrity and behavior, please consult the college bulletin (also available online).

Outline of Topics:

TOPICS

Introduction to Proofs

* Why do Mathematicians Prove Things?
* What Kinds of Statements are to be Proved?
* Definitions, Axioms, Theorems, and Proofs

Logic

2.1. Statements
2.2. The Negation of a Statement
2.3. The Disjunction and Conjunction of Statements
2.4. The Implication
2.5. More On Implications
2.6. The Biconditional
2.7. Tautologies and Contradictions
2.8. Logical Equivalence

Direct Proof and Proof by Contrapositive

3.1. Trivial and Vacuous Proofs
3.2. Direct Proofs
3.3. Proof by Contrapositive
3.4. Proof by Cases

More on Direct Proof and Proof by Contrapositive

4.1. Proofs Involving Divisibility of Integers
4.2. Proofs Involving Congruence of Integers
4.3. Proofs Involving Real Numbers

4.4. Proofs Involving Sets
4.5. Fundamental Properties of Set Operations
4.6. Proofs Involving Cartesian Products of Sets

Proof by Contradiction
5.2 Proof by Contradiction
5.3 A Review of Three Proof Techniques

Mathematical Induction
6.1 The Principle of Mathematical Induction
6.2 A More General Principle of Mathematical Induction

Equivalence Relations
8.1 Relations
8.2 Properties of Relations
8.3 Equivalence Relations
8.4 Properties of Equivalence Classes
8.5 Congruence Modulo *n*
8.6 The Integers Modulo *n*

Functions
9.1 The Definition of Function
9.3 One-to-one and Onto Functions
9.4 Bijective Functions
9.5 Composition of Functions
9.6 Inverse Functions

Cardinalities of Sets
10.1 Numerically Equivalent Sets
10.2 Denumerable Sets
10.3 Uncountable Sets
10.4 Comparing Cardinalities of Sets

To complete all three of the following chapters is **optional**. At least **one must be included**:

Proofs in Number Theory
11.1 Divisibility Properties of Integers
11.2 The Division Algorithm
11.3 Greatest Common Divisor
11.4 The Euclidean Algorithm
11.5 Relatively Prime Integers
11.6 The Fundamental Theorem of Arithmetic

Proofs in Calculus
12.1 Limits of Sequences
12.2 Infinite Series
12.3 Limits of Functions
12.4 Fundamental Properties of Limits of Functions
12.5 Continuity
12.6 Differentiability

Proofs in Group Theory
13.1 Binary Operations

13.2 Groups
13.3 Permutation Groups
13.4 Fundamental Properties of Groups
13.5 Subgroups
13.6 Isomorphic Groups

Note: * denotes a section based on supplementary material.

Suggeted Calendar of topics:

Week 1	Introduction to proofs, Logic: 2.1-2.3
Week 2	Logic: 2.4-2.8
Week 3	Direct Proofs and Proofs by Contrapositive: 3.1-3.4
Week 4	More on Direct Proofs and Proofs by Contrapositive: 4.1-4.3
Week 5	More on Direct Proofs and Proofs by Contrapositive: 4.4-4.6
Week 6	Exam 1; Proof by Contradiction: 5.2, 5.3
Week 7	Mathematical Induction: 6.1, 6.2
Week 8	Equivalence Relations: 8.1-8.6
Week 9	Functions: 9.1, 9.3-9.6
Week 10	Cardinalities of Sets: 10.1-10.4
Week 11	Exam 2; Proofs in Number Theory: 11.1-11.3
Week 12	Proofs in Number Theory: 11.4-11.6
Week 13	Proofs in Calculus: 12.1-12.3
Week 14	Proofs in Calculus: 12.4-12.6
Week 15	Final Exam