Borough of Manhattan Community College
Mathematics Department

SAMPLE PROBLEMS: EXEMPTION FROM MAT 056

The examination to be exempt from MAT 056 will consist of twenty questions, which will be drawn from the following topics.

Literal equations  
Radicals
Quadratic equations  
Simplifying fractions
Absolute value equations  
Adding fractions
Simultaneous equations  
The equation of a straight line
Inequalities  
The slope of a straight line
Word problems  
Trigonometry of a right triangle
Rational exponents  
The laws of logarithms

Here are some representative questions.

1. Solve for $x$: \[ \frac{ax}{c} + b - d = e \]
   
a) \[ x = \frac{ce - b + d}{a} \]  
b) \[ x = \frac{c(e + b - d)}{a} \]  
c) \[ x = \frac{c(e - b + d)}{a} \]  
d) \[ x = ac(e - b + d) \]  
e) \[ x = \frac{e + c(d - b)}{a} \]

2. \[ x^\frac{3}{2} = x^2 x^\frac{3}{2} \]
   
a) \[ x^\frac{5}{3} \]  
b) \[ x^\frac{3}{4} \]  
c) \[ x^\frac{5}{2} \]  
d) \[ x^\frac{1}{5} \]  
e) \[ x^\frac{3}{5} \]

3. Add these fractions: \[ \frac{2}{x^2 - 9} + \frac{3}{x^2 - 4x + 3} \]
   
a) \[ \frac{5x + 7}{(x - 3)(x + 3)(x - 1)} \]  
b) \[ \frac{5x + 11}{(x - 3)(x + 3)(x - 1)} \]  
c) \[ \frac{5}{(x - 3)(x + 3)(x - 1)} \]  
d) \[ \frac{5x - 11}{(x - 3)(x + 3)(x - 1)} \]  
e) \[ \frac{5}{(x^2 - 9)(x^2 - 4x - 3)} \]
4. \[ \frac{a}{\frac{a}{b} - \frac{a}{c}} = \]

   a) \( \frac{abc}{b - c} \)
   b) \( \frac{bc}{c - b} \)
   c) \( \frac{bc}{b - c} \)
   d) \( \frac{a^2 (b - c)}{bc} \)
   e) \( \frac{bc}{a (c - b)} \)

5. In the right triangle below, angle \( \theta = 36^\circ \), and side \( b = 10 \) cm. Find side \( a \).
   Use the following:
   \[
   \begin{align*}
   \sin 36^\circ &= .588 & \tan 36^\circ &= .727 \\
   \cos 36^\circ &= .809 & \cot 36^\circ &= 1.38
   \end{align*}
   \]

5. a) 5.88 cm  
   b) 8.09 cm  
   c) 1380 cm  
   d) 13.8 cm  
   e) 7.27 cm

Answers:
1. c  
2. e  
3. a  
4. b  
5. d
1. Simplify: $7x - 5[5x - 3(4x + 2) - 1]$
2. Solve for $p$: $S = pr^3 + prs$
3. Solve the following equation: $\frac{5}{6}x - \frac{3}{4}(6x + 5) + 1 = \frac{2}{3}(9x - 5) + 5$
4. Solve the following absolute value inequality and graph your solution set: $2|x + 5| \geq 14$
5. Solve the following equation be sure to check the solutions: $\frac{10}{x^2 - 25} - \frac{1}{x - 5} = \frac{3}{x + 5}$
6. Solve: $\frac{5}{x^2} + 5 = \frac{26}{x}$
7. Write down a quadratic equation that has roots: $\{\frac{5}{2}, -5\}$
8. Find the $x$ and $y$ intercepts of the line $5x - 4y = 30$, then graph.
9. What is the equation of the line that passes through the point $(1,1)$ and is perpendicular to the line $y = -2x + 3$?
10. What is the equation of the line that passes through the point $(3,-1)$ and is parallel to the line $y = 5x + 2$?
11. What is the slope of the line expressed by the equation $\frac{x}{2} + \frac{y}{8} = 1$
12. Subtract the following complex numbers. Then, write the answer in standard form: $(3 - 4i) - (2 - i)$ (Hint: answer in $a + bi$)
13. Multiply the complex numbers and then write the answer in standard form: $(4+2i)(5-3i)$
14. Express the following as a single fraction in simplified form: $\frac{1 + \frac{1}{x - 2}}{\frac{3}{x + 2}}$
15. Express the following as a single fraction in simplified form: $\frac{ab}{\frac{1}{b} + \frac{1}{a}}$
16. Solve for $x$: $15x^2 + 11x - 6 = 0$
17. Simplify the following. Express your answer using only positive exponents. $\left(\frac{3p^3q^{-4}}{p^{-2}q^{-5}}\right)^{-2}$

Revised: 4/11/11
18. Solve the inequality \( -(a+1)-4 \leq 2a-8 \). Graph solution on the number line.

19. Solve the system of equations \[
\begin{align*}
-6x - 10y &= 20 \\
3x + 5y &= 25
\end{align*}
\]

20. Michael has $1.85 in dimes and nickels. He has a total of 25 coins. How many of each does he have?

21. Simplify the following completely: \( 32^5 \).

22. Solve for \( x \): \( \sqrt{3x + 10} = x \)

23. Express \( \frac{2}{\sqrt{3} - 1} \) as an equivalent fraction with a rational denominator in simplified form.

24. Rationalize the denominator in the expression: \( \frac{\sqrt{2} + \sqrt{3}}{2 - \sqrt{3}} \).

25. Find the sum and simplify your answer completely: \( \frac{x}{x^2 + 5x + 6} + \frac{x}{x^2 - 9} \)

26. Divide and simplify your answer completely: \( \frac{2x^2 + 7x + 3}{x^2 - 16} + \frac{4x^2 + 8x + 3}{2x^2 - 5x - 12} \)

27. Write the expression \( "5 \log_a x - \frac{1}{4} \log_a y + 2 \log_a z" \) as a single logarithm.

28. Find the exact value of \( \tan 150^\circ \).

29. Find the length of side \( c \) in a triangle labeled \( \Delta ABC \), if \( a=6 \) cm, \( b=10 \) cm and \( C=60^\circ \).

30. A student stands at ground level, 600 feet away from a building. When she looks up to the top of the building, she finds that the angle of elevation is 60°. What is the height of the building? Express your answer in the simplest radical form.

31. Solve \( y = \log_{27} \left( \frac{1}{3} \right) \) by writing it in exponential form first.

32. If \( \sin \theta = \frac{3}{5} \) and \( \theta \) terminates in quadrant II, find \( \tan \theta \).

33. Given that \( \sin(25^\circ) \approx 0.4226 \), \( \cos(25^\circ) \approx 0.9063 \), and \( \tan(25^\circ) \approx 0.4663 \), find the length of the side labeled \( x \) in the triangle below:

![Triangle Diagram]

Revised: 4/11/11
1. $42x + 35$
2. $\frac{5}{x^2 + 5x} = p$
3. $x = -\frac{53}{116}$
4. $x \geq 1$ or $x \leq -6$

$\quad -6 \quad \quad 1$

5. No solution
6. $x = 5, \frac{1}{5}$
7. Any multiple of: $(x - \frac{5}{2})(x + 5)$ or $(2x - 5)(x + 5)$
8. $x$-intercept is 6, $y$-intercept is $-\frac{7}{2}$

9. $y = \frac{1}{2}x + \frac{1}{2}$
10. $y = 5x - 16$
11. $m = -4$
12. $1 - 3i$
13. $26 - 2i$
14. $\frac{x+2}{x-3}$
15. $\frac{a+b}{(ab)^2}$
16. $\frac{-11 \pm \sqrt{481}}{30}$
17. $\frac{1}{q^3 + q^2}$
18. $a \geq 1$
19. No solution
20. 12 dimes and 13 nickels
21. $16$
22. $x = 5$
23. $\sqrt{3} + 1$
24. $2\sqrt{2} + 2\sqrt{3} + \sqrt{6} + 3$
25. $\frac{x^2 + 4x - 6}{(x+3)(x+2)(x-3)}$
26. $\frac{x+3}{x+4}$
27. $\log_b \frac{x^2 + 2}{\sqrt{y}}$
28. $-\frac{\sqrt{3}}{3}$
29. $2\sqrt{19}$
30. $600\sqrt{3} \text{ ft.}$
31. $y = -\frac{1}{3}$
32. $\tan \theta = -\frac{3}{4}$
33. $x \approx 7.4608$

Revised: 4/11/11
Inequalities and Absolute Value Equation

I. Solve each linear inequality and graph the solution set.

1) \(3x - 7 < 2\)  
2) \(-3x - 7 < 2\)

Solve each double inequality and graph the solution set

3) \(5 < 3x - 7 \leq 8\)  
4) \(3 + x \leq 3x + 1 < 2x + 7\)

Solve the quadratic inequalities and graph the solution set

5) \(x^2 - x - 6 > 0\)  
6) \(x^2 + 2x < 3\)

Solve the rational inequalities and graph the solution set

7) \(\frac{x^2 - x - 2}{x^2 - 4x + 3} \leq 0\)
8) \(\frac{4}{x + 5} > \frac{1}{2x + 3}\)

II. Solve the absolute value equations below and graph each.

9) \(|3x - 7| = 2\)  
10) \(|x - 2| < 7\)
11) \(|\frac{2x + 3}{2}| > 5\)

12) \(|2x| = |x - 3|\)  
13) \(|x + 2| \geq |x + 1|\)

14) Express the double inequality \(-3 \leq x \leq 7\) as an absolute value inequality.

15) The perimeter of a rectangle is to be between 180 inches and 200 inches. What is the range of values for its length if the width is to be 40 inches?

16) One acute angle of a right triangle is not less than 30° more than twice the other acute angle. What are the possible measures of the larger angle?

17) The perimeter of a square is to be between 20 meters and 60 meters. What is the range of values for its area?

(\(^*\)) Optional for 056 students
Answers:

1. $x < 3$
2. $x > -3$
3. $4 < x \leq 5$
4. $1 \leq x < 6$
5. $x \in (-\infty, -2) \cup (3, \infty)$
6. $-3 < x < 1$
7. $x \in (-1, 1) \cup (2, 3)$
8. $x \in (-5, \frac{3}{2}) \cup (-1, \infty)$
9. $x = \frac{5}{3}, 3$
10. $-5 < x < 9$
11. $x \in (-\infty, -\frac{13}{2}) \cup \left(\frac{7}{2}, \infty\right)$
12. $x = -3, 1$
13. $[-\frac{3}{2}, \infty)$
14. $|x - 2| \leq 5$
15. $50 < L < 60$
16. $70 \leq x < 90$
17. $25 < A < 225$
Exponents and Polynomials Review Sheet

1. Simplify and express each answer using positive exponents only.
   a) \((-2x^3y)^2\) \((-3x^{-2}y^2)^3\)
   b) \((2x^3y^{-2}z^0)^2\) \(\div 8x^{-3}y^2\)

2. Simplify each expression, if possible.
   a) \(5\sqrt{75x^2} - 2\sqrt{12x^2}\)
   b) \(\sqrt{x^2y} + \sqrt{8x^2y} - \sqrt{200x^2y}\)

3. Simplify by performing the indicated operations
   a) \((x^3 + 3x^2 + 3x - 1) - (x^2 + 2x + 1)\)
   b) \((2x^3y^2 - 5xy + x^2y^3) + (3xy - x^2y^3) - (x^3y^2 + 2xy)\)
   c) \((x^2 + x + 9) \cdot (x^2 - 3x - 4)\)

4. Factor using GCF
   a) \(21x^4y - 14x^5y^2\)
   b) \(8x^2 \cdot (x - 1) + 4x (x - 1) = 0\)

5. Factor by Grouping
   a) \(ax^2 + 15 - 5ax - 3x\)
   b) \(2 - y^2 + 2x - xy^2\)

6. Factor and solve each quadratic trinomial
   a) \(4 - 5y + y^2 = 0\)
   b) \(15x^2 + 7x - 8 = 0\)

7. Factor each trinomial
   a) \(x^4 - x^2 - 12\)
   b) \(6x^5y - 3x^3y^2 - 30xy^3\)

8. Factor as the sum and difference of two squares.
   a) \(25x^2 - 144\)
   b) \(9a^2 - 121b^2\)

9. Factor as the difference of two cubes. Solve if possible.
   a) \(125x^3 - 64 = 0\)
   b) \(8x^3 + 343 = 0\)

10. Factor completely
    a) \(81x^4 - 256y^4\)
    b) \(12x^2y + 22xy^2 - 60y^3\)

Remember:

- Perfect square trinomials \((a \pm b)^2 = a^2 \pm 2ab + b^2\)
- The difference of two squares \(a^2 - b^2 = (a - b)(a + b)\)
- The difference of two cubes \(a^3 - b^3 = (a - b)(a^2 + ab + b^2)\)
- The sum of two cubes \(a^3 + b^3 = (a + b)(a^2 - ab + b^2)\)
Answers

1. a) $-108y^8$  
   b) $\frac{x^9}{2y^6}$

2. a) $21\sqrt{3}x$  
   b) $x\sqrt{y} - 8x\sqrt{2y}$

3. a) $x^3 + 2x^2 + x - 2$  
   b) $x^3y^2 - 4xy$  
   c) $x^4 - 2x^3 + 2x^2 - 31x - 36$

4. a) $7x^4y(3 - 2xy)$  
   b) $4x(x - 1)(2x + 1)$, solutions $x = -1/2, 0, 1$

5. a) $(x - 5)(ax - 3)$  
   b) $(2 - y^2)(x + 1)$

6. a) $(y - 4)(y - 1) = 0$  
   sol. $y = 1, y = 4$  
   b) $(15x - 8)(x + 1)$  
   sol. $x = 8/15, x = -1$

7. a) $(x - 2)(x + 2)(x^2 + 3)$  
   b) $3xy(x^2 + 2y)(2x^2 - 5y)$

8. a) $(5x - 12)(5x + 12)$  
   b) $(3a - 11b)(3a + 11b)$

9. a) $(5x - 4)(25x^2 + 20x + 16) = 0$, only one real solution: $x = \frac{4}{5}$  
   b) $(2x + 7)(4x^2 - 14x + 49) = 0$, only one real solution: $x = \frac{-7}{2}$

10. a) $(3x - 2y)(3x + 2y)(9x^2 + 4y^2)$  
    b) $2y(3x + 10y)(2x - 3y)$
Rational Expressions. Word Problems.

1. A man can construct a wall in 20 hrs. With the help of his apprentice assistant the task would take 12 hrs. How long would it take the apprentice, working alone, to construct the wall?
   a) 35 hrs   b) 25 hrs   c) 30 hrs   d) 40 hrs

2. One water pipe can fill a tank in 90 min while a second pipe can fill the tank in 60 min. How long would it take to fill the tank if both pipes were used?
   a) 36 min   b) 30 min   c) 42 min   d) 48 min

3. A jet can fly 550 mph in calm air. Traveling with the wind, the plane can fly 2520 mi in the same amount of time as it flies 1880 mi against the wind. Find the rate of the wind.
   a) 55 mph   b) 80 mph   c) 70 mph   d) 60 mph

4. A canoeist can paddle at a rate of 8 mph in still water. Traveling with the current, the canoe traveled 30 mi in the same amount of time as it traveled 18 mi against the current. Find the rate of the current.
   a) 2.5 mph   b) 1 mph   c) 3 mph   d) 2 mph

5. The rate of a motorcycle is 40 mph greater than the rate of a bicycle. The motorcycle travels 150 mi in the same amount of time as the bicycle travels 30 mi. Find the rate of the motorcycle.
   a) 50 mph   b) 10 mph   c) 40 mph   d) 55 mph

6. One number is twice another. The sum of their reciprocals is \( \frac{1}{2} \). The value of the largest number is:
   a) 4   b) 6   c) 10   d) 8

7. Solve the equation \( 1 - \frac{1}{x} = \frac{6}{x^2} \).
   a) -3 & 2   b) 3 & 2   c) 3 & -2   d) -3 & -2

8. If a certain number is added to the numerator and denominator of \( \frac{9}{11} \), the result is \( \frac{7}{10} \). Find the number.
9. Two people working together can do a job in 3 hrs. How long will it take the slower person to do the same job if one of them is 3 times as fast as the other?
   a) 1 hr  b) 4 hr  c) 1/3 hr  d) 12 hrs

10. The formula $P = \frac{1}{a} + \frac{1}{b}$ is used by optometrists to help determine how strong to make the lenses for a pair of eyeglasses. If $a = 10$ and $b = 0.2$, the corresponding value of $P$ is:
   a) $\frac{5}{1}$  b) 10.2  c) $6\frac{1}{1}$  d) 5.1

Solving Quadratic and Cubic Equations. Practice Sheet.

1. solve by the quadratic formula: $3x^2 = x + 4$

2. Solve for $a$: $(2a - 3)^2 = -16$

3. Solve for $x$: $x^2 - 8 = 0$

4. Solve by the quadratic formula: $2x + 3 = -2x^2$

5. Solve by completing the square: $(x + 5)(x - 3) = -25$

6. Solve by any method: $\frac{2a}{a - 2} + \frac{2}{a + 1} = 0$

7. Solve by completing the square: $2y^2 - 6y + 4 = 0$

8. a) Use the discriminant to identify the number and kind of solutions to:
   
   $9 - 12x = -4x^2$

   b) Solve for $x$. 

9. Find $\lambda$ so that $3x^2 = \lambda x - 12$ has one rational solution.

10. Find an equation that has $x = 3$ and $x = \frac{1}{2}$ as solutions.

11. Solve for $x$: $x^4 - 6x^2 - 27 = 0$ (Show all solutions).

12. Solve for $x$: $x - 8\sqrt{x} + 12 = 0$.

13. Solve for $x$: $(x - 1)^2 + 2(x - 1) - 35 = 0$

14. Solve and graph the solution set for $x$: $9 \geq x^2$

15. Solve for $x$: $x^3 + 27 = 0$
Answers  TOPIC #7

1. \(-1, \frac{4}{3}\)

2. \(\frac{3}{2} \pm 2i\)

3. \(2, -1 \pm \sqrt{3}i\)

4. \(\frac{-1 \pm i \sqrt{5}}{2}\)

5. \(-1 \pm 3i\)

6. \(-1 \pm \sqrt{3}\)

7. \(1, 2\)

8. a) One rational solution
   
   b) \(\frac{3}{2}\)

9. \(\lambda = \pm 12\)

10. \(2x^2 - 7x - 3 = 0\)

11. \(-3, 3, -\sqrt{3}i, \sqrt{3}i\)

12. \(4, 36\)

13. \(-6, 6\)

14. \(-3 \leq x \leq 3\)

15. \(-3, \frac{3}{2} (1 \pm \sqrt{3}i)\)
Exponential & Logarithmic Relations Practice Sheet

1) Write each of the following expressions in logarithmic (log) form
   a) \( 16 = 4^2 \)  b) \( 2^{-5} = \frac{1}{32} \)  c) \( \left( \frac{1}{3} \right)^{-2} = 9 \)  d) \( 10^{-3} = .001 \)

2) Write each expression in exponential form
   a) \( \log_2 64 = 6 \)  b) \( \log_2 8 = 3 \)  c) \( \log_{10} .001 = -3 \)  d) \( \log_3 \frac{1}{81} = -4 \)

3) Solve each expression for \( x \)
   a) \( \log_3 x = 2 \)  b) \( \log_3 27 = x \)  c) \( \log_{25} 5 = -x \)  d) \( \log_5 x = -3 \)

4) Simplify each of the following
   a) \( \log_{10} 1000 \)  b) \( \log_3 (\log_8 6) \)  c) \( \log_{25} 5^3 \)  d) \( \log_4 [\log_2 (\log_2 16)] \)

5) Find the pH of a bottle of vinegar, if the concentration of the hydrogen ion is \( [H^+] = 10^{-3} \).

6) If an earthquake has a magnitude of 8 on the Richter Scale, how many times greater is its shockwave than the smallest shockwave measurable on a seismograph?

7) Write each expression as a single logarithm
   a) \( 4 \log_2 x + 5 \log_2 y \)  b) \( 3 \log_{10} x - \log_{10} y - \log_{10} z \)
   c) \( 2 \log_3 x - 3 \log_3 y^2 \)  d) \( \frac{1}{2} \log_{10} x - \frac{3}{4} \log_{10} y - \frac{4}{5} \log_{10} z \)

8) Solve each of the following equations
   a) \( \log_3 x + \log_3 3 = 1 \)  b) \( \log_8 x + \log_8 (x - 1) = 1 \)
   c) \( \ln (x - 6) = 0 \)  d) \( \log_5 \sqrt{x} - \log_5 \sqrt{6x - 5} = 0 \)
9) Evaluate using a calculator

a) \( \log 21 \)    b) \( \ln 0.0462 \)    c) \( \log 23,400 \)    d) \( \ln 100 \)

10) If $400 is deposited in an account that earns 10% annual interest compounded twice a year, how much money will be in the account after 5 years?

Note: \( A = P \left[ 1 + \frac{r}{n} \right]^{nt} \)  \( pH = -\log [H^+] \)  \( M = \log_{10} T \)

Answers:

1. a) \( \log_4 16 = 2 \)  b) \( \log_2 \frac{1}{32} = -5 \)  c) \( \log_3 9 = -2 \)  d) \( \log .001 = -3 \)

2. a) \( 2^6 = 64 \)  b) \( 2^3 = 8 \)  c) \( 10^{-3} = .001 \)  d) \( 3^{-4} = \frac{1}{81} \)

3. a) 9  b) 3  c) \( -\frac{1}{2} \)  d) \( \frac{1}{125} \)

4. a) 3  b) 0  c) \( \frac{3}{2} \)  d) \( \frac{1}{2} \)

5. \( pH = 3 \)

6. \( 10^8 \)

7. a) \( \log_2 x^4 y^5 \)  b) \( \log_{10} \frac{x^3}{yz} \)  c) \( \log_3 \frac{x^2}{y^6} \)  d) \( \log_{10} \frac{x^{\frac{1}{3}}}{y^{\frac{1}{4}} z^{\frac{1}{5}}} \)

8. a) 1  b) 3  c) 7  d) 1

9. a) 1.322  b) -3.075  c) 5.369  d) 4.605

10. $651.56

2. Compute the height of the Empire State Building using the information given in illustration 2.

3. A plane is flying at an altitude of 5120 feet. As it approaches an island, the navigator determines the angles of depression as in illustration 3. What is the length of the island in feet?

4. Bill and Paula, standing on the same side of and in line with the Washington Monument, are looking at its top. The angle of elevation from Bill's position is 34.1°, and the angle of elevation from Paula's position is 60.0°. If Bill and Paula stand on level ground and are 500 feet apart, how tall is the monument?

5. Refer to illustration 4 and find \( \theta \).

6. Use the information given in Illustration 5 to compute the height of the Gateway Arch in St. Louis.

7. Refer to Illustration 4 and find \( \phi \).

8. Compute the height of the Sears Tower using the information given in Illustration 6 below.

9. Use the information given in Illustration 7 below to compute the height of the figure part of the Statue of Liberty.
Answer:

1. 60.654 ft
2. 1250.048 ft
3. 5890.776 ft
4. 555.776 ft
5. 48.317°
6. 631.213 ft
7. 117.525°
8. 1453.139 ft
9. 150.843 ft
In the triangle above the angle $\alpha$ has the triangular relationships:

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<thead>
<tr>
<th>Primary</th>
<th>Secondary</th>
<th>Formally</th>
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<td>$\sin (\alpha) = \frac{opp}{hyp}$</td>
<td>$\cosecant (\alpha) = \left(\sin(\alpha)\right)^{-1}$</td>
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